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THERMAL CONVECTION IN A POROUS MEDIUM WITH HORIZONTAL CRACKS

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Abstract—This paper is the second part of a study of thermally-driven convection in a porous medium composed of very thin layers alternating with thick layers of a different material. While in the first part [*Int. J. Heat Mass Transfer* **26**, 761–780 (1983)] the thin layers had very small permeability (sheets), here the effects of thin, highly permeable horizontal layers (cracks) are investigated. Even and odd numbers of layers are studied separately, with a variety of results being given for the critical Rayleigh number and the subsequent heat transport for slightly supercritical convection. Calculated streamline patterns indicate that local, or small-scale, convection is generally absent for the present problem. In the earlier paper, it was shown that a sheet tended to 'close' a constant-pressure upper surface—it is found here that, conversely, a crack in contact with an impermeable boundary tends to 'open' that surface. Odd numbers of layers give either zero or two cracks in contact with the boundaries—in the latter case there is good convergence towards homogeneous anisotropy as the number of layers increases. The results have application to, for example, insulation problems where gaps can occur between successively applied layers of insulating materials.

NOMENCLATURE

c	specific heat of saturating fluid
d	layer thickness
g	gravitational acceleration
k	thermal conductivity of saturated medium
K	permeability
L	cell width
M	number of cracks
N	number of layers in system
Nu	Nusselt number
Ra	Rayleigh number
Ra^*	Rayleigh number based on vertical parameter K_V
ΔT	temperature difference across system.

Greek symbols

α	thermal expansion coefficient
β, β'	permeability ratios, K_2/K_1, K_1/K_2
ε, ε'	layer thickness ratios, $d_2/(d_1+d_2)$, $d_1/(d_1+d_2)$
ν	kinematic viscosity of saturating fluid
ξ	permeability anisotropy, K_H/K_V
ρ_a	reference density of saturating fluid
σ	slope coefficient for Nusselt number.

Subscripts

c	critical value
H	horizontal
i	ith layer
thin	thin layer
min	minimum value with respect to variation over L
V	vertical.

1. INTRODUCTION

IN SPITE of its technical importance and relevance to geothermal problems, thermal convection in porous media composed of discrete layers of different materials with constant permeabilities has only recently been investigated theoretically. McKibbin and O'Sullivan [1] developed a method for treating an arbitrary number of layers and solved the onset problem for two and three layers. In a following paper [2], they extended the analysis to estimate the heat transfer for slightly supercritical convection. Masuoka *et al.* [3] had earlier studied convection currents for the two-layer configuration.

A theory of thermal convection in multilayered porous media based on the methods of McKibbin and O'Sullivan [1, 2] has followed—two papers by the present authors have been published [4, 5], with the investigation centring on the case of alternating layers. This is of basic interest because as the number of layers increases, there is a convergence towards homogeneous anisotropy [6], i.e. the stratified medium is able to be modelled increasingly closely by a homogeneous layer with an equivalent anisotropy in permeability.

The first paper concentrated on the case of equal layer thicknesses [4]. Now, as in the last study [5], the case where every alternate layer is very thin is investigated. If a very thin stratum cutting through an otherwise homogeneous porous material is to have any effect, its permeability must be either very small ('sheet' limit) or very large ('crack' limit). The sheet limit of very small permeability was discussed in ref. [5]. Here, the opposite limit of thin layers with very large permeability is considered. The word 'crack' will be associated with this limit, because large permeability requires relatively large, well-connected pores. The thin layers might consist of fluid only, except for a skeletal network needed to prevent deformation of the matrix. As will be seen, the flow in such thin layers is almost horizontal (i.e. along the cracks).

In addition, the present study has a connection with the general problem of thermal convection in a mixed configuration of both a porous medium and a Newtonian fluid [7, 8]. Whereas it was found in the first part of this investigation that a sheet can close a permeable boundary [5], it will be shown here that a crack can open an impermeable surface.

The sheet and crack limits indeed have very different properties. In the sheet limit there is a very strong preference for local convection [3], which gives nearly square-formed cells in the internal, more permeable layers. Such a layer may become locally unstable when the local Rayleigh number reaches a critical value (dependent on the geometry) somewhat below $4\pi^2$. A sufficient condition for this local convection to begin was given in ref. [4, equation (6.1)].

By contrast, local convection generally does not occur in the crack limit. Consequently, this form of discrete layering is more reliably modelled by a homogeneous anisotropic analogue. It is important for such modelling that local convection be avoided, as it causes a serious breakdown of the analogue. However, there is one peculiar feature of the crack limit which has no counterpart in the anisotropic model: there is a strong horizontal flow in the cracks. As will be seen, this may cause slow convergence towards the analogue for the heat transport when there are any nearly immobile layers next to the boundaries.

The distinction between even and odd numbers of layers has been of only minor interest in previous parts of this work [4, 5], in which only even numbers of layers were studied explicitly. For the crack limit, however, the case of odd numbers of layers will also be investigated, as it is connected with many interesting effects.

As in ref. [5], the mathematical procedure of McKibbin and O'Sullivan [1, 2] will be followed. A brief review of this procedure was given in ref. [5, Appendix].

2. FORMULATION OF THE CRACK LIMIT PROBLEM

A fluid-filled horizontal slab with layering in permeability only is considered. The thermal conductivity of all parts of the saturated medium k, is assumed constant. A general formulation for free thermally-driven convection in a multilayered porous medium [2] is summarized in a previous paper [5, Appendix]. As in ref. [5], a relative layer thickness ratio ε is introduced, defined by

$$\varepsilon = \frac{\min\{d_1, d_2\}}{d_1 + d_2}, \quad (1)$$

where d_1 and d_2 are the thicknesses of the alternating strata, numbered from the base of the system. This definition is convenient because the crack limit then always corresponds to a small value of ε.

The effective average horizontal and vertical permeabilities of the system of alternating layers are given by

$$K_H = \sum_{i=1}^{N} \frac{d_i}{d} K_i, \quad (2)$$

$$K_V = 1 \Big/ \sum_{i=1}^{N} \frac{d_i}{dK_i}, \quad (3)$$

where d_i and K_i are the thickness and permeability of layer i ($i = 1, 2, \ldots, N$), with N being the total number of layers making up the total thickness d.

Now, as was pointed out previously [5], in the 'sheet' limit for an even number of layers, one sheet is without significance, namely the one in contact with an impermeable boundary. The thin layer limit case of N (even) layers treated there covers $N-1$ (odd) layers as well. In the present case, however, each layer has considerable significance, whether as a permeable crack or as a thick layer of the surrounding medium. Consequently, all layers are now incorporated in the definitions of K_H and K_V given above.

For an even number of layers, with thick and thin strata occurring in pairs throughout the system, the average directional permeabilities are given by

$$K_H = \frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}, \quad (4)$$

$$K_V = \frac{d_1 + d_2}{d_1/K_1 + d_2/K_2}, \quad (5)$$

and are indeed independent of N, the total number of layers. For an odd number of layers, both K_H and K_V depend on N. Although slightly more complex than equations (4) and (5), the expressions are easily derived and will not be given here.

The effective anisotropy of the layered system is measured by the parameter ξ defined by

$$\xi = K_H/K_V. \quad (6)$$

For a given configuration of layers, the criterion for instability of the fluid is given by the critical value Ra_c^* of a (scaled) Rayleigh number Ra^* defined in terms of K_V by

$$Ra^* = \frac{\rho_a g c \alpha K_V \Delta T d}{4\pi^2 \nu k}, \quad (7)$$

where ΔT is the overall temperature difference across the system, and the other parameters have their usual meanings and are defined in the nomenclature. For the critical value ΔT_c, a periodic convection pattern of cell width L begins to form. For slightly higher values of ΔT, and thus of Ra^*, the average vertical heat flux, measured by the Nusselt number, Nu, can be expressed by

$$Nu = 1+\sigma\left(\frac{Ra^*}{Ra^*_c}-1\right), \tag{8}$$

where σ depends on the system configuration. The critical value Ra^*_c, and the corresponding value of the slope parameter for the heat transport σ both vary with cell width L. Ra^*_c takes its minimum value $Ra^*_{c,\min}$ when $L = L_c$ [5, Appendix].

Onset of convection and slightly supercritical flow in a system confined between horizontal, impermeable, perfectly conducting (and thus isothermal) plane surfaces will be studied here. Relevant results for a homogeneous porous medium so confined and with permeability anisotropy ξ may be found in Kvernvold and Tyvand [6] and are

$$Ra^*_{c,\min} = \frac{1}{4}(1+\xi^{-1/2})^2, \tag{9}$$

$$L_c = \xi^{1/4}, \tag{10}$$

$$\sigma = 2.0. \tag{11}$$

These values will form the basis for comparison when a homogeneous anisotropic analogue is sought for the multilayered systems.

3. CONVERGENCE TOWARDS THE CRACK LIMIT

Before considering the possible use of a homogeneous layer with equivalent anisotropy as an analogue for the layered system with cracks, the convergence towards the crack limit, as the more permeable layers become very thin, is investigated. This limit depends on whether the number of layers N is even or odd, and on which of layers 1 and 2 is a crack. Let M denote the number of cracks in the porous medium, while N is the total number of layers. Then there are three separate cases to consider:

Case I: even number of layers. One crack is in contact with a boundary. In this case $N = 2M$.

Case II: odd number of layers, with thick layers at the top and bottom. No crack is in contact with a boundary, i.e. the cracks are internal only. Then $N = 2M+1$.

Case III: odd number of layers, with a crack in contact with each of the top and bottom boundaries. In this case, $N = 2M-1$.

The appropriate limiting values of ξ as ε becomes small, in Table 1, are given in terms of the relative thickness parameter ε defined in equation (1) and the layer permeability ratio $\beta = K_2/K_1$. For convenience, when layer 1 is the thin more permeable layer, the values of ε and β are written as ε' and $\beta' = K_1/K_2$.

For the study of this convergence as $\varepsilon \to 0$ (i.e. as

Table 1. Limiting values of the effective permeability anisotropy $\xi = K_H/K_V$ as the layer thickness ratio ε becomes very small. N is the total number of layers

	N even	N odd
$\varepsilon = d_2/(d_1+d_2) \ll 1$	$1+\varepsilon\beta$	$1+\frac{N-1}{N+1}\varepsilon\beta$
$\beta = K_2/K_1 \gg 1$		
$\varepsilon' = d_1/(d_1+d_2) \ll 1$	$1+\varepsilon'\beta'$	$1+\frac{N+1}{N-1}\varepsilon'\beta'$
$\beta' = K_1/K_2 \gg 1$		

min $\{d_1, d_2\} \to 0$), the method is different to that used for the sheet problem [5]. There, a special sheet parameter was defined, and kept constant as ε (and ξ) varied. Here, ε will be varied with ξ kept constant. This gives a simultaneous presentation of the convergence towards the crack and sheet limits, as $\varepsilon \to 0$ and $\varepsilon \to 1$, respectively. In between, $\varepsilon = 0.5$ corresponds to the case of equal layer thicknesses which was studied in ref. [4].

In Figs. 1–4: (a) shows the critical value $Ra^*_{c,\min}$ of the Rayleigh number, minimized over cell width; (b) shows the corresponding cell width L_c; and (c) shows the corresponding slope parameter σ for the heat transfer, as defined in equation (8). These quantities are displayed as functions of the relative thickness ε (or ε') for different numbers of layers. Figures 1 and 2 show results for even numbers of layers (case I) with $\xi = 2$ and 10, respectively. Figure 3 shows results for an odd number of layers with the less permeable layers occurring at the top and the bottom (case II); in the limit as $\varepsilon \to 0$, no cracks are in contact with the upper and lower boundaries—there are internal cracks only—while in the limit $\varepsilon \to 1$, two impermeable sheets are in contact with the boundaries. Figure 4 shows results for an odd number of layers with the more permeable layers at the top and the bottom (case III); in the limit $\varepsilon' \to 0$, cracks are in contact with both boundaries, while as $\varepsilon' \to 1$, only internal sheets are formed. Both sets of results in Figs. 3 and 4 are for $\xi = 10$.

In Figs. 1–4, the limits as $\varepsilon \to 1$ are in principle known from the previous study on impermeable sheets [5, Fig. 5], because a sheet in contact with an impermeable boundary has negligible influence. For instance, the limits as $\varepsilon \to 1$ for the lowest values of N in Figs. 1–3 are

$$(Ra^*_{c,\min}, L_c, \sigma) = (\xi^{-1}, 1, 2). \tag{12}$$

That there is no unique value of ε which gives optimal convergence towards anisotropy is clearly shown by Figs. 1–4. (The results for homogeneous anisotropy are included as broken lines in these figures—these are straight lines because ξ is constant.) For instance, with $\xi = 2$ and N even, minimal deviations from the results for homogeneous anisotropy for $Ra^*_{c,\min}$, L_c and σ appear to take place for $\varepsilon \cong 0.73$, $\varepsilon \to 0$ and $\varepsilon \to 1$, respectively (see Fig. 1). This means that the homogeneous analogue is favourable for the crack limit

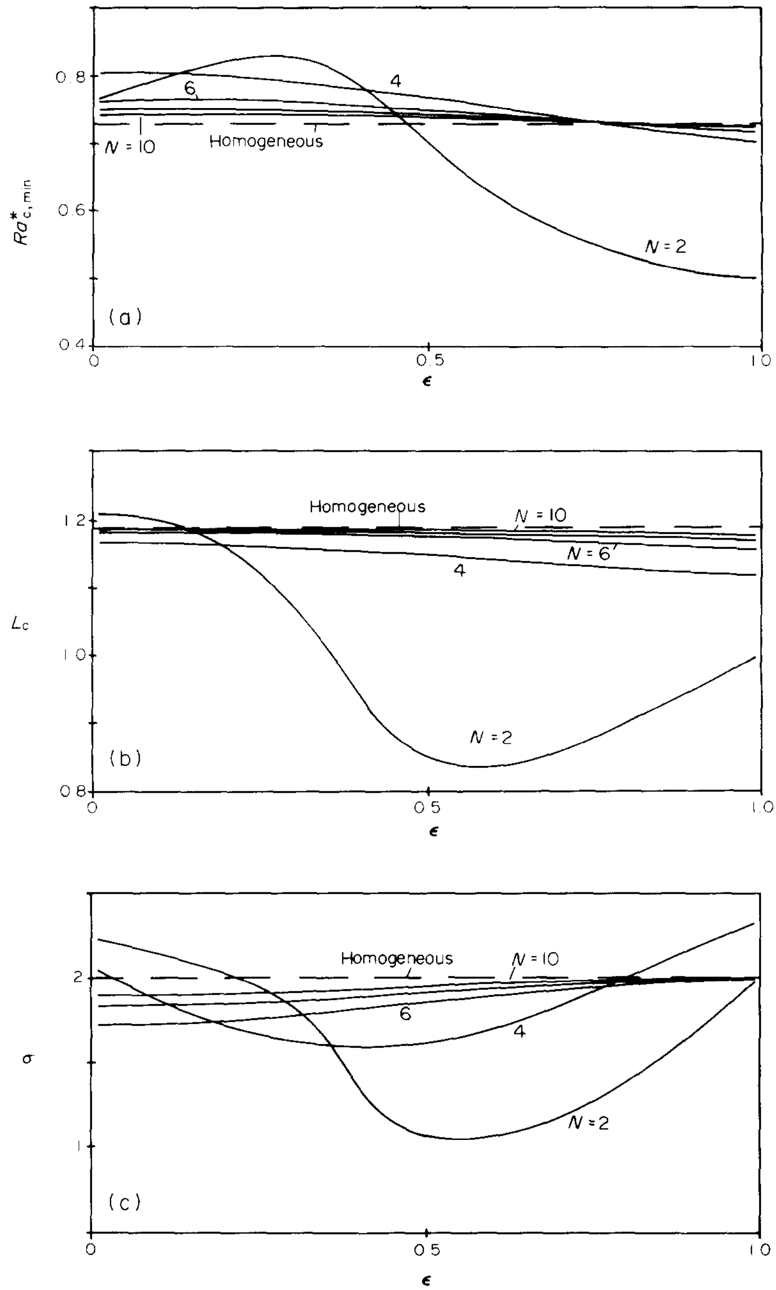

FIG. 1. Variation of critical Rayleigh number $Ra^*_{c,\min}$, cell width L_c and corresponding values of the Nusselt number slope parameter σ, with the layer thickness ratio $\varepsilon = d_2/(d_1+d_2)$, for a system composed of an even number of layers, N, for permeability anisotropy $\xi = 2$: (a) $Ra^*_{c,\min}$ vs ε; (b) L_c vs ε; (c) σ vs ε.

only in estimating the cell width for the convection patterns. However, for $\xi = 10$ (Fig. 2), the optimal value for cell width estimation has changed to approximately $\varepsilon = 0.1$.

For a given value of effective anisotropy ξ, local convection never occurs in the crack limit $\varepsilon \to 0$; it is a case, then, for which the anisotropic analogue may be reasonably applied. However, for larger values of ε, Figs. 2–4 show dramatic examples of local convection. There are discontinuities in the curves for L_c and σ and in the slope of the curves for $Ra^*_{c,\min}$, with large-scale convection to the left and local convection to the right of each discontinuity. As ξ increases, the number of layers for which local convection can occur increases, and the values of ε corresponding to the discontinuities decrease. So, although there is no local convection in the crack limit $\varepsilon \to 0$, it may occur quite close to this limit (i.e. for small ε) when ξ is sufficiently large. This means that, when the crack limit is investigated, ε must be carefully chosen to be small enough so that local convection is avoided—such a choice is far more important here than in the study of the sheet limit [5].

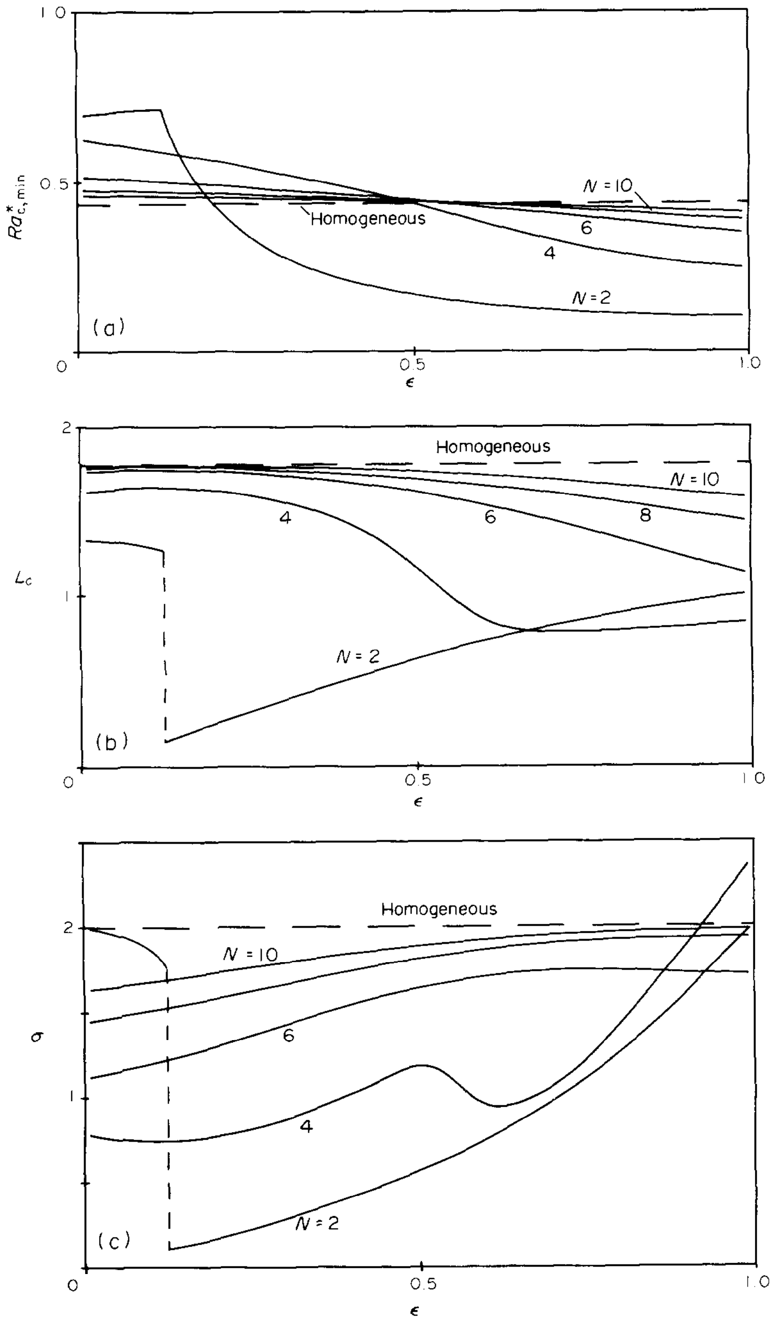

FIG. 2. Variation of $Ra^*_{c,min}$, L_c and corresponding values of σ with ε, for N even and $\xi = 10$: (a) $Ra^*_{c,min}$ vs ε; (b) L_c vs ε; (c) σ vs ε.

Now, local convection must be preferred when the local, or layer, Rayleigh number Ra_{thin} for the thin stratum, defined by

$$Ra_{thin} = \frac{\rho_a g c \alpha d_{thin} K_{thin} \Delta T_{thin}}{\nu k},$$

exceeds $4\pi^2$ before large-scale convection begins. (As pointed out in ref. [4], the value $4\pi^2$ is an upper limit; the destabilizing effect of bounding interfaces which are not isothermal or impermeable lowers this value.) This gives a sufficient condition for local convection, in terms of previously defined values and for small ε, as follows:

$$\left.\begin{aligned} Ra^*_{c,min} &> \frac{N^2}{4}\frac{1}{\varepsilon(\xi-1)} \quad \text{for case I,} \\ Ra^*_{c,min} &> \frac{N^2-1}{4}\frac{1}{\varepsilon(\xi-1)} \quad \text{for cases II and III.} \end{aligned}\right\} \quad (13)$$

These are sufficient conditions only; local convection may occur outside these ranges. As will be shown in Figs. 12–14(a), $Ra^*_{c,min}$ is always larger than 0.25;

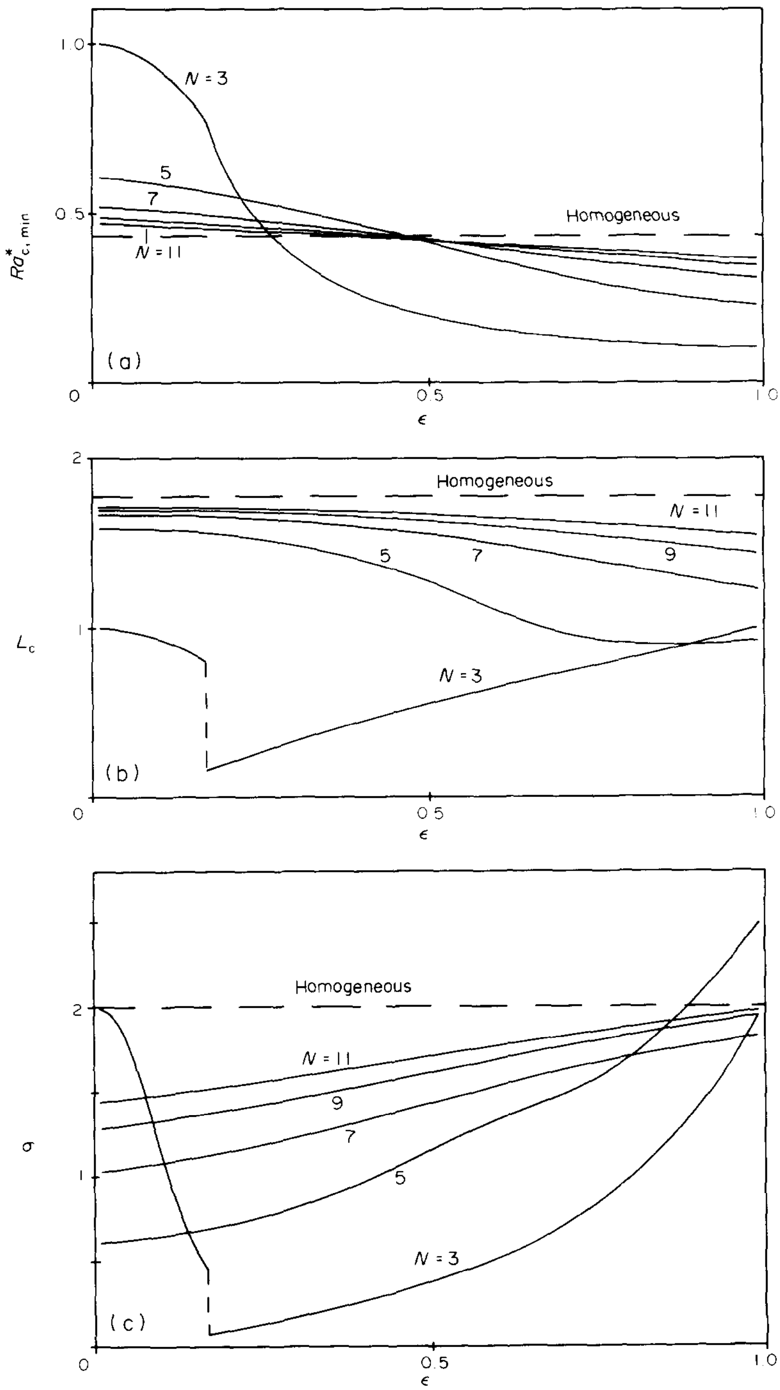

FIG. 3. Variation of $Ra^*_{c,\min}$, L_c and corresponding values of σ with ε, for an odd number of layers N with less permeable layers on the bottom and top, and $\xi = 10$: (a) $Ra^*_{c,\min}$ vs ε; (b) L_c vs ε; (c) σ vs ε.

accordingly even weaker sufficient conditions may be obtained from the above by replacing $Ra^*_{c,\min}$ by 0.25, to obtain, in rearranged form

$$\left.\begin{aligned} \varepsilon &> \frac{N^2}{\xi-1} \quad \text{for case I,} \\ \varepsilon &> \frac{N^2-1}{\xi-1} \quad \text{for cases II and III.} \end{aligned}\right\} \quad (14)$$

Local convection is not relevant in the crack limit $\varepsilon \to 0$, so solutions for such convection are not sought. Representative mathematical solutions for large-scale convection when ε is small, but finite, are found below. When inequality (14) is satisfied, for large values of ξ, local convection may occur. However, since Figs. 1–4 indicate that, for a given value of ξ, a small but finite value of ε provides a good approximation to the solutions for large-scale convection in the limit $\varepsilon \to 0$,

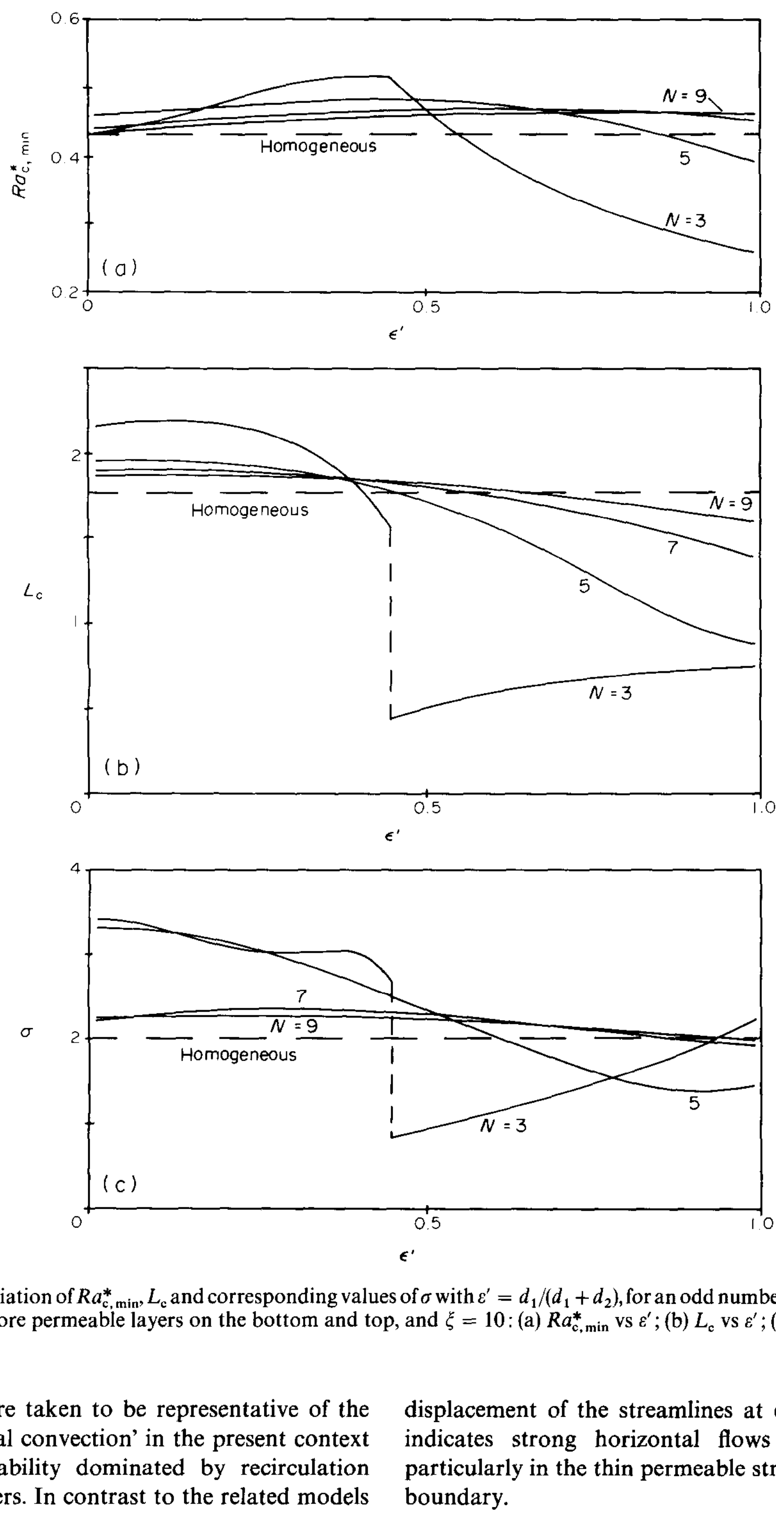

FIG. 4. Variation of $Ra^*_{c,\min}$, L_c and corresponding values of σ with $\varepsilon' = d_1/(d_1+d_2)$, for an odd number of layers N with more permeable layers on the bottom and top, and $\xi = 10$: (a) $Ra^*_{c,\min}$ vs ε'; (b) L_c vs ε'; (c) σ vs ε'.

such solutions are taken to be representative of the crack limit. 'Local convection' in the present context denotes an instability dominated by recirculation within single layers. In contrast to the related models treated before [4, 5], this does not necessarily imply a small preferred cell width.

Some examples of the flow patterns which occur at onset of convection in systems with thin very permeable layers are shown in Figs. 5–10. Streamline patterns for an even total number of layers (case I), when $N = 2, 4, 6, 8, 10$ and $\varepsilon = 0.02$ are shown in Figs. 5 and 6, for $\xi = 2$ and 10, respectively. Large-scale convection is clearly seen in all of these cases, with almost horizontal displacement of the streamlines at each crack. This indicates strong horizontal flows in the cracks, particularly in the thin permeable stratum next to the boundary.

Examples of streamline patterns for case II, where there is an odd number of layers, but the cracks are internal only, are shown in Figs. 7 and 8. These configurations are symmetric with respect to top and bottom. For only one internal crack ($N = 3$) with a relative thickness $\varepsilon = 0.01$, Fig. 7 shows the effect of increasing ξ. As the permeability of the crack becomes larger relative to the outside layers, the flow becomes 'local' in the sense that it is concentrated near the more

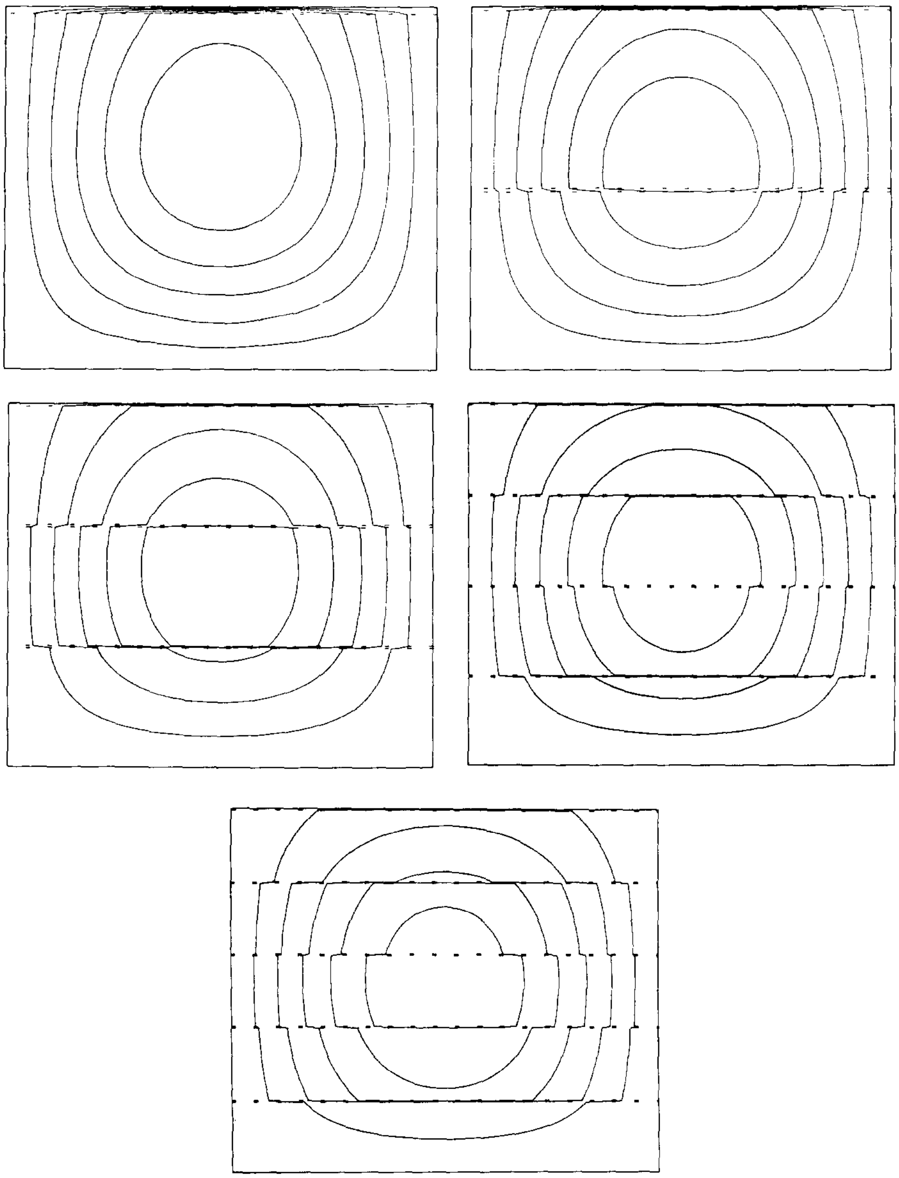

FIG. 5. Streamlines at onset of convection for an even number of layers $N = 2, 4, 6, 8, 10$ (1, 2, 3, 4, 5 cracks) when $\varepsilon = 0.02$ and $\xi = 2$.

permeable middle layer. Nevertheless, even up to $\xi = 1000$, the cell width still corresponds to a 'large-scale' type of convection. (The streamlines correspond to divisions of $\psi_{max}/6$—for $\xi = 100$, for example, approximately half the flow is circulating through the less permeable layers.) For large numbers of layers, shown in Fig. 8 where $\xi = 10$, $\varepsilon = 0.01$ and $N = 5, 7, 9, 11$ (2, 3, 4, 5 internal cracks) only the two outside less permeable layers tend to become inert, with large-scale convection in the rest of the system. Although the heat transport is reduced due to these less active layers, as N increases they become thinner relative to the total thickness and have less effect on the heat transport—σ therefore increases with N, as is seen in Fig. 3(c).

When there are cracks next to both boundaries (case III), it is expected that there will be strong horizontal flow in both of these thin layers. That this is indeed so may be seen in Figs. 9 and 10. When there are no internal cracks, i.e. $N = 3$, as shown in Fig. 9, most of the fluid passes directly from the lower crack where it is heated, to the upper crack where it is cooled, and consequently the heat transport is high. The high value of σ for $\xi = 10$ is shown in Fig. 4(c). For larger numbers of cracks, some examples being shown in Fig. 10 for $N = 5, 7, 9$ when $\varepsilon = 0.01$ and $\xi = 10$ the flow patterns are similar, with strong horizontal flow in the cracks, especially those in contact with the external boundaries.

The particular case $N = 2$ is studied in Fig. 11. When ξ is not much greater than 1, the streamlines meet the more permeable layer at sharp angles, and there is a strong recirculation of fluid within the less permeable

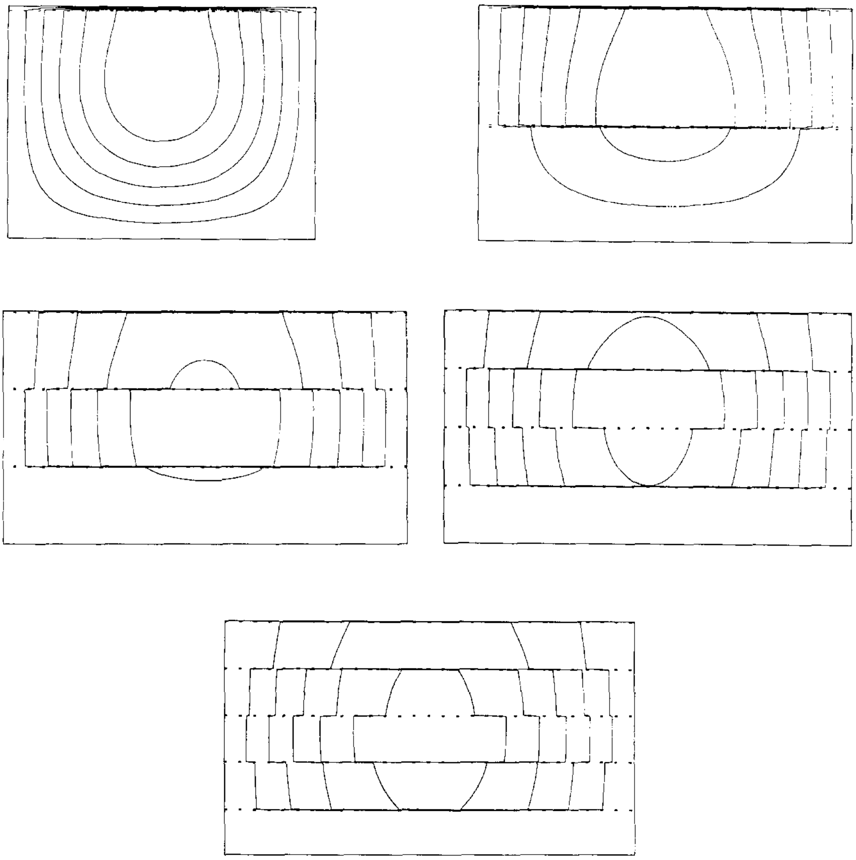

FIG. 6. Streamlines at onset of convection for an even number of layers $N = 2, 4, 6, 8, 10$ when $\varepsilon = 0.02$ and $\xi = 10$.

thick layer. However, as ξ becomes large, all the streamlines crossing the boundary into or out of the crack meet it at right angles. The interface thus behaves as a constant pressure surface or 'open' top—indeed the streamline patterns for $\xi \geqslant 10$ are seen to be almost identical to that for a homogeneous layer with an open top. From McKibbin and Tyvand [4, Table 1], the relevant limit values are

$$(Ra^*_{c,\min}, L_c, \sigma) = (0.686, 1.35, 1.95).$$

It should perhaps be noted that in this limit case there is still some small amount of recirculation within the thick layer, being 6.17% of the total volume flux [9].

Some of the curves for the heat transport parameter σ in Figs. 1–4, have rather complicated behaviour. It was found in ref. [5] that local convection patterns separated by one internal impermeable sheet are an efficient way of transporting heat. This is because the fluid moves in opposite directions on each side of the sheet. This effect, as $\varepsilon \to 1$, is seen in Figs. 1–4 by the curves for $N = 4, 4, 5$ and 3 layers, respectively. When ε is smaller, but local convection still persists, the heat transport is considerably smaller because of the existence of almost immobile internal layers—this is also evident from these curves.

In particular, the curves for σ are quite complex for a small number of layers. As may be seen from the streamline patterns in Figs. 4–10, internal cracks guide the flow, so that in the crack limit $\varepsilon \to 0$, a passive layer may form near a boundary, impeding the heat transport (although, as mentioned above, this effect decreases as the number of layers increases and the passive layers become relatively thinner). This again indicates the local stabilizing effect of a boundary [4, p. 331]—the thick layer next to an impermeable surface is less active than the other thick internal layers, even though they all have the same local Rayleigh number. Conversely, a crack next to an impermeable boundary becomes very active compared to internal cracks which are near the middle of the layered system.

4. RESULTS FOR THE CRACK LIMIT

Finally, the ability of the equivalent homogeneous anisotropic analogue [6] to model the layered system is investigated.

In Figs. 12–14, the basic results are displayed as functions of ξ for cases I, II and III, respectively. The crack limit is represented by choosing $\varepsilon = 0.01$. In Figs. 12–14: (a), (b) and (c) show the critical Rayleigh number $Ra^*_{c,\min}$, the corresponding cell width L_c and the heat flux slope parameter σ, respectively, with broken lines indicating the values for a homogeneous layer with

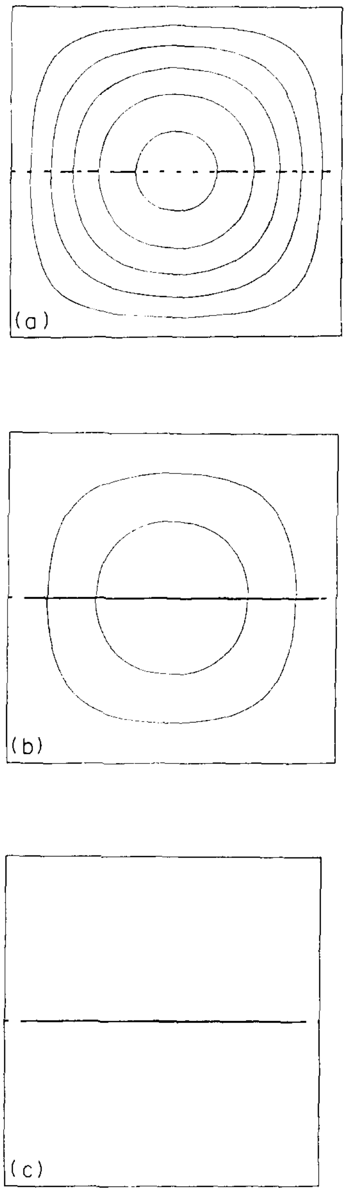

FIG. 7. Streamlines at onset of convection for a medium with one internal crack ($N = 3$) of relative thickness $\varepsilon = 0.01$, for: (a) $\xi = 10$; (b) $\xi = 100$; (c) $\xi = 1000$.

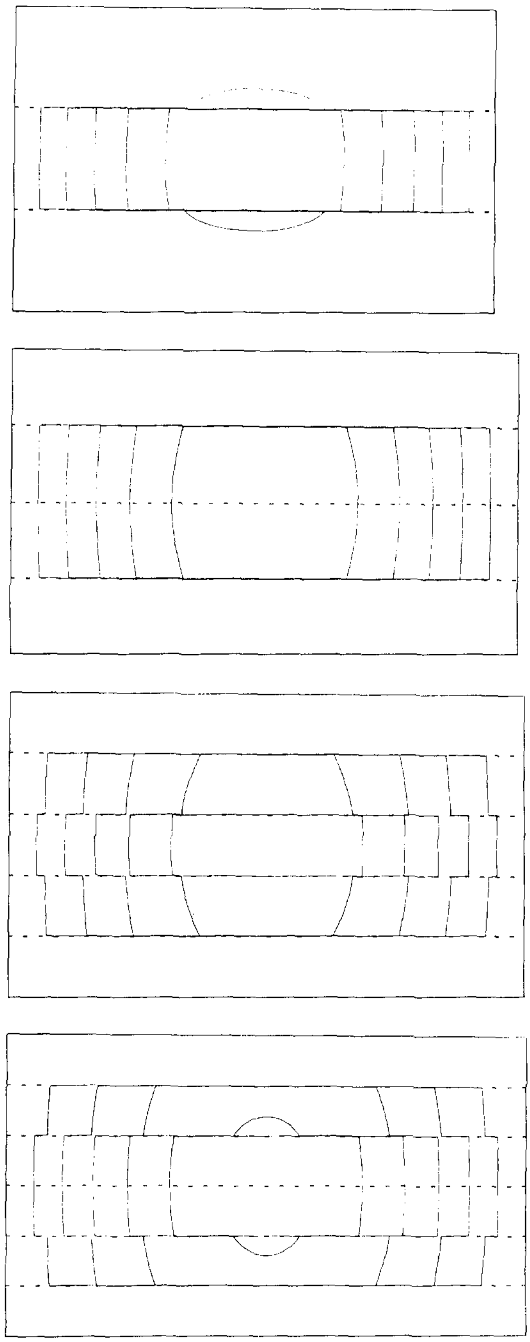

FIG. 8. Streamlines at onset of convection for an odd number of layers with 2, 3, 4, 5 internal cracks of relative thickness $\varepsilon = 0.01$, for $\xi = 10$.

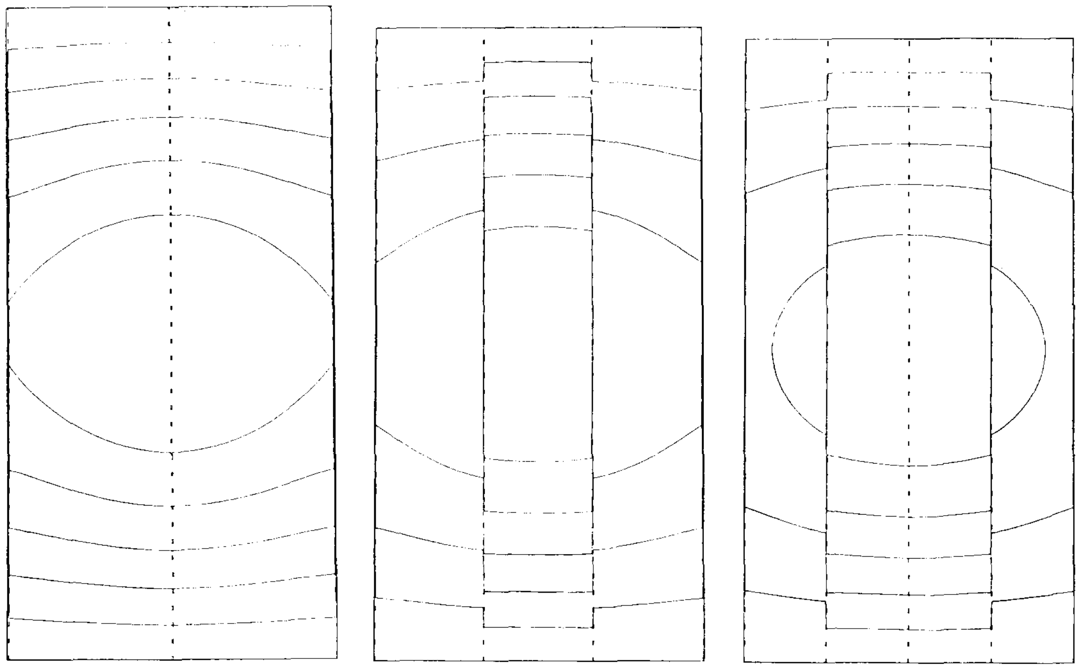

FIG. 10. Streamlines at onset of convection for an odd number of layers with cracks at each boundary; $N = 5, 7, 9$ (3, 4, 5 cracks) for $\varepsilon = 0.01$ and $\xi = 10$.

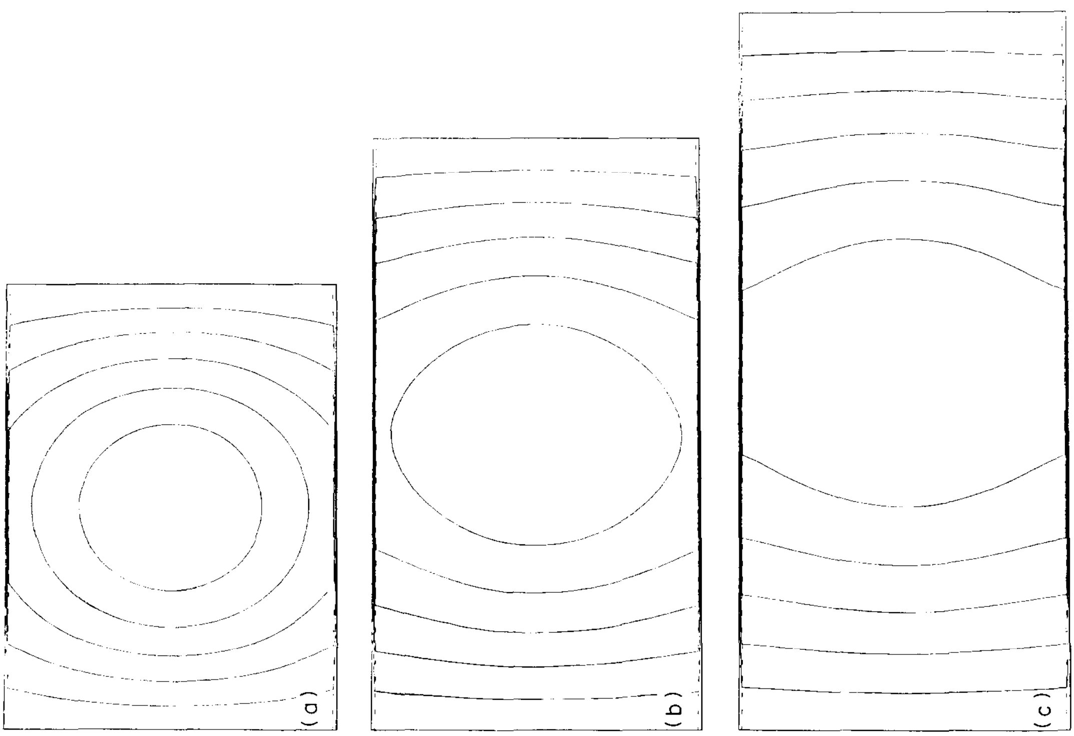

FIG. 9. Streamlines at onset of convection for a medium with a crack of relative thickness $\varepsilon = 0.01$ next to each boundary ($N = 3$), for: (a) $\xi = 2$; (b) $\xi = 5$; (c) $\xi = 10$.

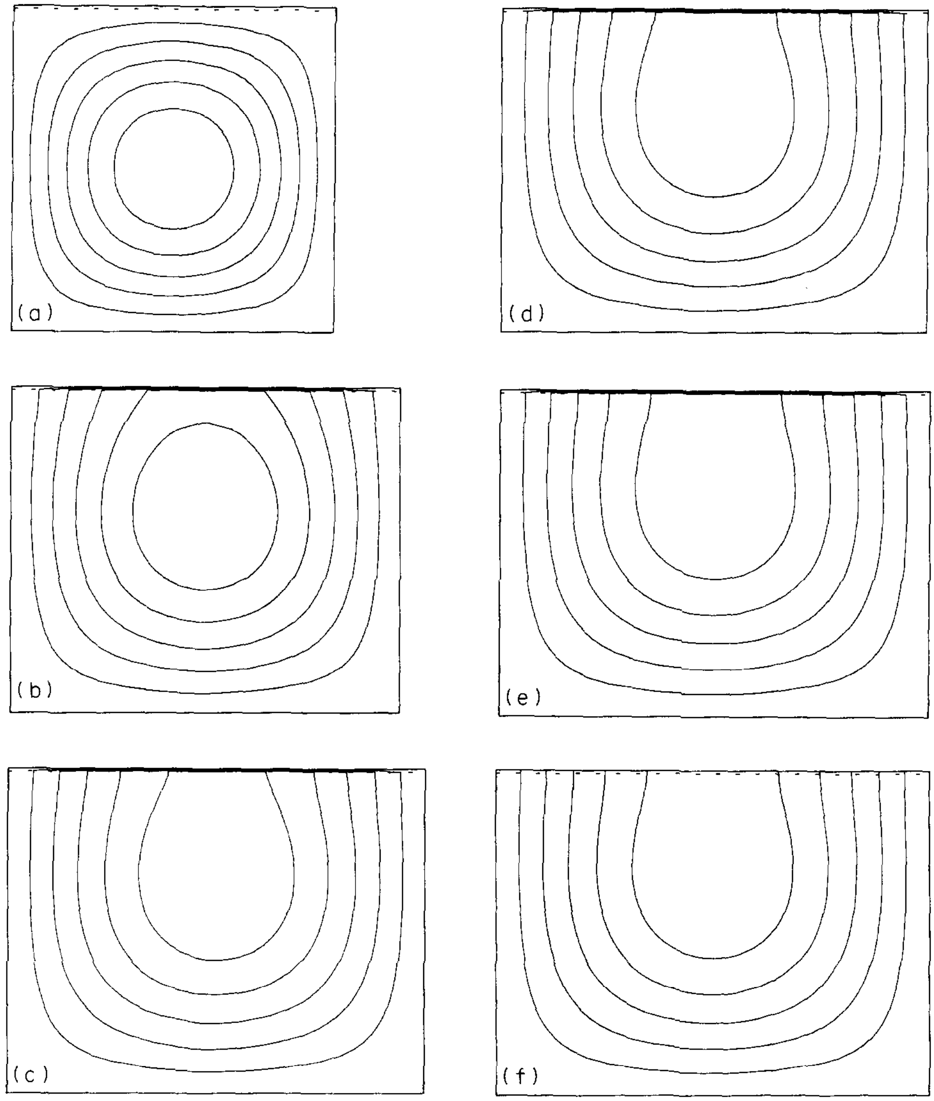

FIG. 11. Onset of convection for the case $N = 2$, with one crack of relative thickness $\varepsilon = 0.01$ next to a boundary. Streamlines are shown for: (a) $\xi = 1$; (b) $\xi = 2$; (c) $\xi = 5$; (d) $\xi = 10$; (e) $\xi = 20$; (f) $\xi = 1$ with constant pressure (open) top boundary condition.

equivalent anisotropy, calculated from equations (9)–(11).

The only case which shows a definite breakdown of the crack limit approximation is for $N = 3$ in Fig. 13 (case II). In condition (13), $Ra^*_{c,\min}$ may be replaced by 1 because one central crack does not influence large-scale convection due to symmetry. Accordingly ε should not be greater than $2/(\xi - 1)$ to represent the crack limit. So for $\varepsilon = 0.01$, a breakdown of the approximation might be expected as ξ approaches about 200, and this is indeed what happens. It is seen most clearly in Fig. 13(c) where σ begins to decrease drastically from the constant crack limit value of 2. The streamline patterns in Fig. 7 show how the convective flow tends to become localized near the crack as ξ becomes large. All other results in Figs. 12–14 appear to give satisfactory representations for large-scale convection for the limit $\varepsilon \to 0$.

In Fig. 12, results for N even (case I) are displayed. There is a gradual convergence towards the homogeneous anisotropic analogue as N increases. The convergence is relatively slow because there is a thick layer near one boundary. In this thick layer the flow is very weak except in the cases where $N = 2$ and 4 with ξ small.

Figure 13 shows results for case II, where only internal cracks occur. In the crack limit, the centremost crack should have no effect at all, due to symmetry. This is observed from the curves for $N = 3$, except, as pointed out above, when ξ becomes too large to be compatible with the choice of ε. For $N > 3$, the thick layers near the boundaries (Fig. 8) tend to insulate and σ is always relatively small. However, for a given number of cracks and ξ large, the heat transport rate is smaller in case I than in case II. This means that *one* thick layer in

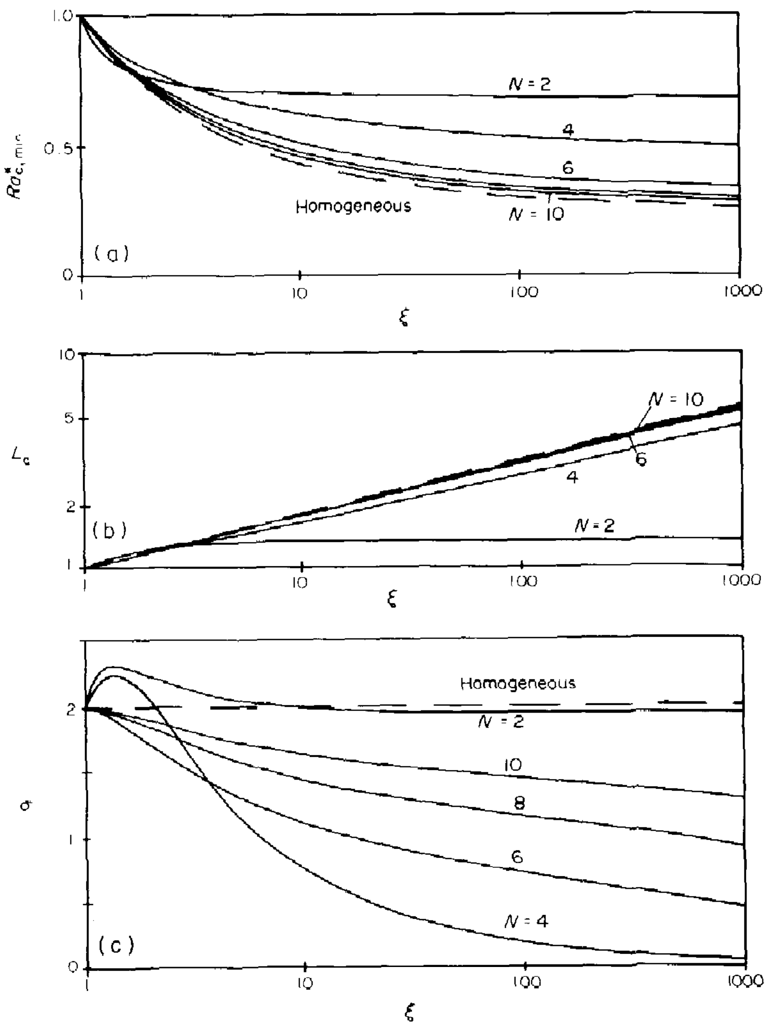

FIG. 12. Variation of $Ra^*_{c,min}$, L_c and corresponding values of σ with ξ for an even number of layers when $\varepsilon = 0.01$. Values are given for $N = 2, 4, 6, 8, 10$ and the homogeneous system : (a) $Ra^*_{c,min}$ vs ξ ; (b) L_c vs ξ ; (c) σ vs ξ.

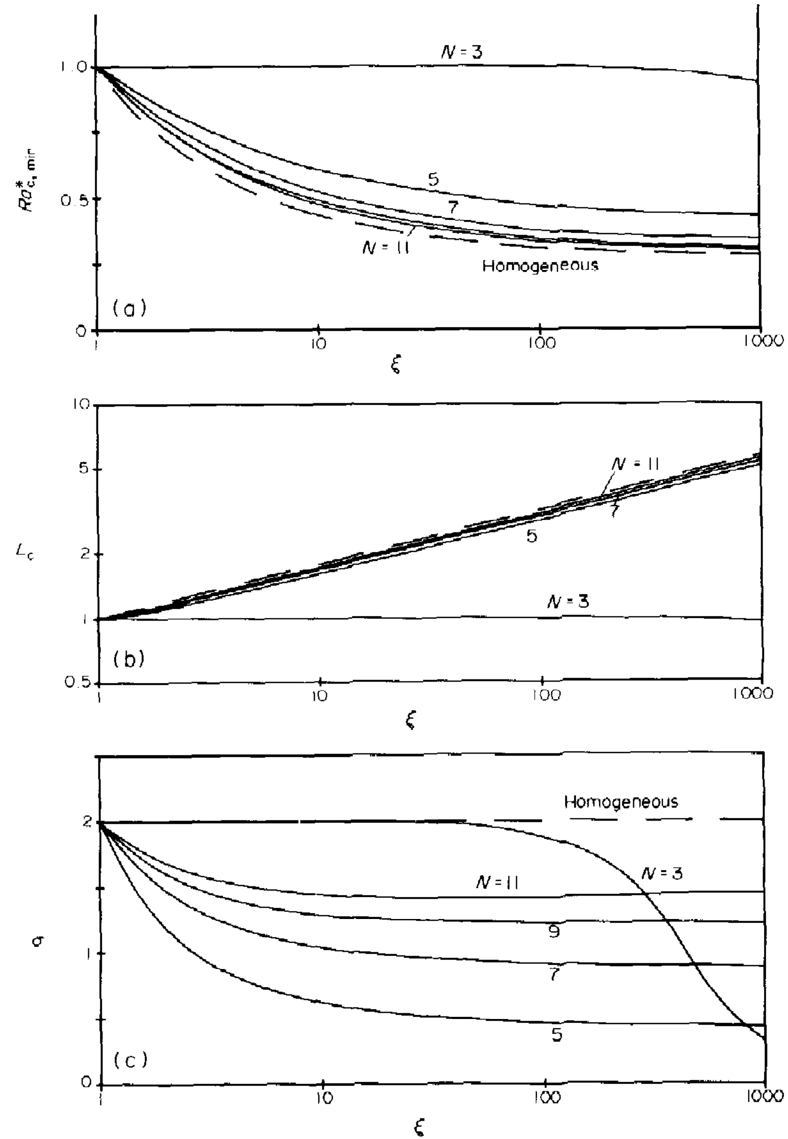

FIG. 13. Variation of $Ra^*_{c,min}$, L_c and corresponding values of σ with ξ for an odd number of layers with internal cracks of relative thickness $\varepsilon = 0.01$. Values are given for $N = 3, 5, 7, 9, 11$ and the homogeneous system : (a) $Ra^*_{c,min}$ vs ξ ; (b) L_c vs ξ ; (c) σ vs ξ.

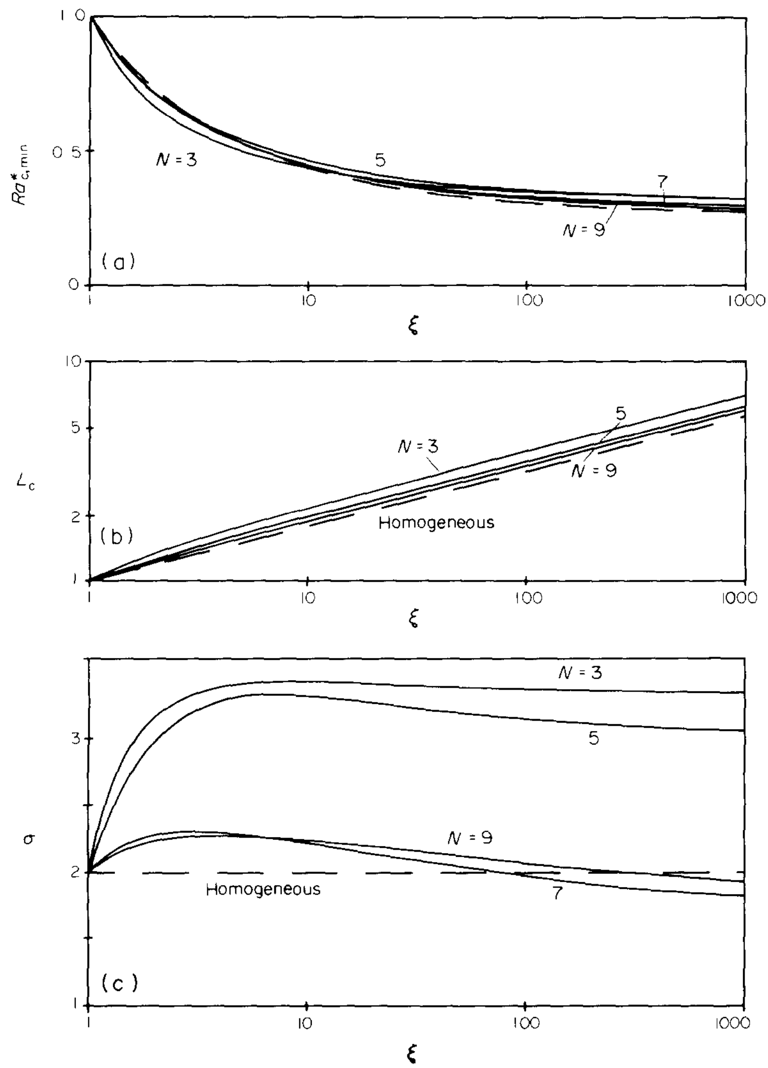

FIG. 14. Variation of $Ra^*_{c,min}$, L_c and corresponding values of σ with ξ for an odd number of layers with cracks at each boundary, when $\varepsilon = 0.01$. Values are given for $N = 3, 5, 7, 9$ and the homogeneous system: (a) $Ra^*_{c,min}$ vs ξ; (b) L_c vs ξ; (c) σ vs ξ.

contact with the boundaries insulates better than *two*. Apart from this, the convergence towards anisotropy is similar in cases I and II (Figs. 12 and 13).

Figure 14 shows that case III has some new properties. When a crack is in contact with each boundary, the flow penetrates all of the layers (see streamline patterns in Figs. 9 and 10). This may give a better convergence towards the homogeneous analogue than for cases I and II where there is only weak flow in the thick layer(s) near the boundaries. From Fig. 14(a) it is observed that, for all values of N, the crack configuration is less stable than the homogeneous limit when $\xi < 3$. This is opposite to the tendency for cases I and II (with the exception $N = 2$, $\xi < 2$ for case I). When $\xi > 10$, cracks give a more stable configuration, although only slightly more so, than the homogeneous case.

From Nield [10], it is known that convection starts at zero Rayleigh number for zero wavenumber in an isotropic medium with two open (constant-pressure) boundaries. It is therefore not surprising that a low critical Rayleigh number and a tendency towards large cell widths is found in case III, since as ξ becomes large, the outermost cracks tend to open the boundaries. Figure 14(b) shows that the critical cell width is always larger than that for homogeneous anisotropy. This is opposite to the trend for cases I and II.

The heat transport in case III is much larger than for the previous cases [see Fig. 14(c)], but not always larger than the homogeneous value $\sigma = 2$. The σ curves are grouped in pairs of N-values, (3, 5) and (7, 9). This is related to the fact that a crack in the middle of the porous medium has a negligible effect on this symmetric configuration (compare streamline patterns in Figs. 9 and 10).

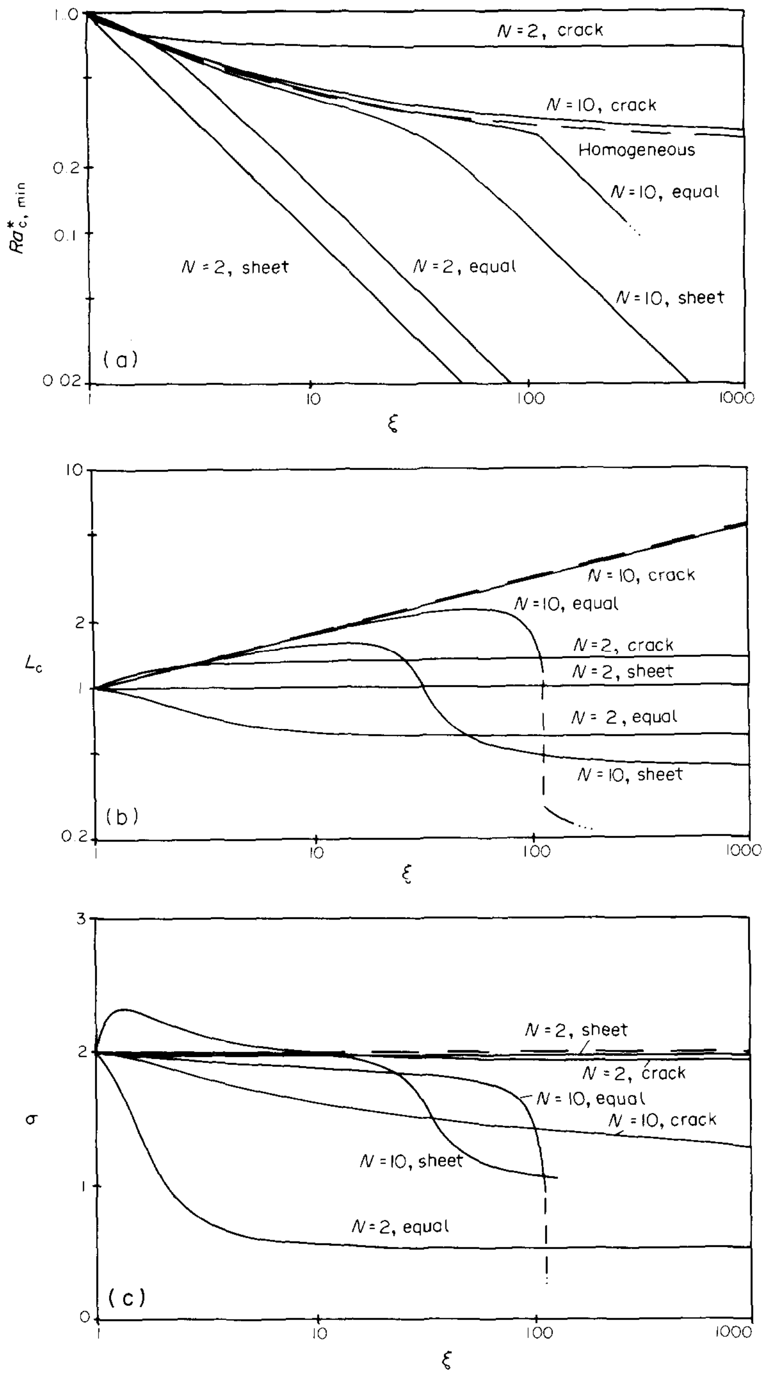

FIG. 15. Variation of $Ra^*_{c,\min}$, L_c and corresponding values of σ with ξ for $N = 2$, 10 and the homogeneous system. Curves are given for the sheet limit and the crack limit (in both cases thicknesses $\varepsilon = 0.01$ are used), for equal layer thicknesses ($\varepsilon = 0.5$) and for the homogeneous anisotropic analogue.

5. SUMMARY AND CONCLUSIONS

The present paper together with the two previous studies by the present authors [4, 5] of convection in layered porous media form a comprehensive set of information. Figure 15 is an attempt to synthesize some of this. It is an explicit comparison between the cases of equal layer thicknesses [4], the sheet limit [5], the present crack limit and the homogeneous anisotropic analogue. Curves for $N = 2$ and 10 are displayed. Not surprisingly, Fig. 15(a) shows that equal layer thicknesses give a critical Rayleigh number that usually lies between those for the sheet and crack limits. For $N = 10$ when the convection is of the large-scale type, a similar conclusion applies to the cell width, shown in Fig. 15(b). For the heat transport parameter, however, the situation is more complicated.

In the sheet limit [5], the effect of a thin layer was only to give *refraction* of the streamlines. This is caused by a discontinuity in the horizontal velocity, due to the pressure drop required to pass through a sheet. In the present crack limit, the effect of a thin layer is a *horizontal displacement* of each streamline, with no pressure drop across a crack. From the streamline figures, it is seen that there is no refraction between meeting streamlines on each side of a crack. However, these neighbouring streamlines usually do not conduct the same fluid particles. The only exception is a crack in the middle of a symmetric configuration, such a crack having no effect at all.

In ref. [5], two different kinematic boundary conditions were applied at the upper surface: impermeable (closed top) and constant pressure (open top). It was found that a sheet is able to 'close' an open top. Here the opposite effect is found: a crack is able to 'open' a closed top.

Using the convention of this study, the critical Rayleigh number for a homogeneous isotropic medium ($\xi = 1$) with a closed top is given by $Ra^*_{c,min} = 1$. Results found here indicate that it is never possible to destabilize a homogeneous porous layer by more than 75% by the introduction of horizontal cracks. No result has been found for which $Ra^*_{c,min}$ is below $\frac{1}{4}$, which is the limit as $\xi \to \infty$, i.e. the medium is infinitely more permeable horizontally than vertically.

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CONVECTION THERMIQUE DANS UN MILIEU POREUX AVEC DE CRAQUELURES HORIZONTALES

Résumé—Ce texte est la seconde partie d'une étude de la convection naturelle dans un milieu poreux composé de très fines couches alternant avec d'autres d'un matériau différent. Alors que dans la première partie [*Int. J. Heat Mass Transfer* **26,** 761–780 (1983)] les couches fines sont de faibles perméabilité, ici on étudie les effets de couches horizontales fortement perméables (craquelures). Des nombres pairs et impairs de couches sont étudiés séparément, avec une variété de résultats donnés pour le nombre de Rayleigh critique ainsi que le transfert thermique correspondant à une convection légèrement supercritique. Des configurations de lignes de courant calculées indiquent que la convection locale, ou de petite échelle, est généralement absente du problème étudié. Dans le texte précédent, on montrait qu'une couche tend à "fermer" une surface supérieure à pression constante, alors qu'ici inversement une craquelure en contact avec une frontière imperméable tend à "ouvrir" cette surface. Des nombres impairs de couches donnent soit zéro soit deux craquelures en contact avec les frontières; dans le premier cas, il y a une bonne convergence vers une anisotropie homogène quand le nombre de couches augmente. Les résultats ont des applications, par exemple, à l'isolation quand des vides se produisent entre des couches contiguës de matériaux isolants.

THERMISCHE KONVEKTION IN EINEM PORÖSEN MEDIUM MIT HORIZONTALEN SPALTEN

Zusammenfassung—Diese Arbeit ist der zweite Teil einer Untersuchung thermisch bedingter Konvektion in einem porösen Medium, das abwechselnd aus sehr dünnen und dicken Schichten aus anderem Material zusammengesetzt ist. Während im ersten Teil [*Int. J. Heat Mass Transfer* **26,** 761–780 (1983)] die dünnen Schichten sehr geringe Durchlässigkeit ("Lamellen") hatten, wird hier der Einfluß dünner, stark durchlässiger horizontaler Schichten (Spalte) untersucht. Gerade und ungerade Schichtzahlen werden getrennt behandelt, man erhält eine Reihe von Ergebnissen für die kritische Rayleigh-Zahl und den folgenden Wärmetransport bei geringfügig überkritischer Konvektion. Die berechneten Stromlinien-Bilder zeigen, daß beim vorliegenden Problem in allgemeinen örtliche oder kleinräumige Konvektion nicht auftritt. In der früheren Arbeit wurde gezeigt, daß eine Lamelle eine "Schließungs-Tendenz" für eine obere Grenzfläche konstanten Drucks bewirkt—hier findet man gegenteilig, daß ein Spalt im Kontakt mit einer undurchlässigen Grenzfläche die Tendenz hat, diese Oberfläche zu "öffnen". Ungerade Schichtzahlen ergeben entweder null oder zwei Spalte in Kontakt mit den Grenzen—im letzteren Fall ergibt sich mit zunehmender Schichtzahl gute Konvergenz in Richtung homogener Anisotropie. Eine Anwendung der Ergebnisse ist beispielsweise bei Isolationsproblemen zu sehen, wo Spalte zwischen nacheinander aufgebrachten Isolationsschichten auftreten können.

ТЕПЛОВАЯ КОНВЕКЦИЯ В ПОРИСТОЙ СРЕДЕ С ГОРИЗОНТАЛЬНЫМИ ТРЕЩИНАМИ

Аннотация—Представлена вторая часть исследования тепловой конвекции в пористой среде, состоящей из тонких слоев материала, чередующихся со слоями другого материала гораздо большей толщины. В то время как в первой части работы [международный журнал "Тепло-и массоперенос" **26**, 761–781 (1983)] рассматривались слои (пластинки) малой толщины с очень небольшой проницаемостью, в данной части исследуются тонкие горизонтальные слои (трещины) с очень высокой проницаемостью. Рассматриваются системы, состоящие из четного и нечетного числа слоев. При этом имеют место различные результаты для критического числа Рэлея и для конвекции при несколько больших его значениях. Полученные линии тока свидетельствуют о том, что в исследуемом случае, как правило, локальная (или мелкомасштабная) конвекция отсутствует. В первой части работы было показано, что слой стремится "закрыть" верхнюю поверхность постоянного давления, здесь же, наоборот, трещина, контактирующая с непроницаемой границей, стремится "открыть" эту поверхность. При нечетном числе слоев с границами контактируют или две трещины или ни одной. В первом случае отмечается пержход к однородной анизотропии по мере увеличения числа слоев. Результаты могут использоваться, например, в задачах, связанных с применением изоляционных материалов, когда возникают зазоры между последовательно наложенными слоями изоляции.